# Core Mathematics C3          For Edexcel
# Advanced Level

## Paper E
## Time: 1 hour 30 minutes

*Instructions and Information*

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

*Advice to Candidates*

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

1. (*a*) Express in the form $Ax + B$, where $A$ and $B$ are to be found,

$$\frac{2x^2 - x - 3}{x + 1} + \frac{x^2 - 4}{x + 2}.$$

**(4)**

(*b*) Hence solve the equation

$$\frac{2x^2 - x - 3}{x + 1} + \frac{x^2 - 4}{x + 2} = x^2 - 9$$

**(2)**

2. Given that $x = \sin^{-1}\left(-\dfrac{3}{5}\right)$, find the value of

(*a*) $\cos x$

**(4)**

(*b*) $\cos 2x$

**(2)**

3. (*a*) Sketch the graph of $y = e^{ax+b}$, given $a$ and $b > 0$.

Mark the coordinates of the point where the graph meets either the $x$-axis or the $y$-axis.

**(2)**

(*b*) Given that when $x = 0$, $y = 4$, find the exact value of $b$.

**(1)**

(*c*) Given further that the gradient at the point whose $x$ coordinate is 2 is equal to $10\,e^5$, find the value of $a$.

**(5)**

4. Differentiate the following functions with respect to $x$.

(*a*) $\cos^2 x$

**(3)**

(*b*) $\dfrac{\ln x}{x}$

**(3)**

(*c*) $x^2 e^x$

**(4)**

5. (*a*) Prove that

$$\sin x + \cot x \cos x = \operatorname{cosec} x.$$

**(3)**

(*b*) Hence or otherwise find the values of $x$, $0 < x < 180°$, which satisfy the equation

$$\cot x \cos x = 3,$$

giving your answers to 1 decimal place.

**(4)**

**6.** Sketch the graphs of $y = \sin x$ and $y = x^3 - 1$, for $0 < x < 2\pi$, indicating clearly the coordinates of the points where the curves meet the axes.

**(4)**

(*a*) Using your graph, explain why there is only one solution to the equation

$$\sin x - x^3 + 1 = 0.$$

**(1)**

(*b*) If $\alpha$ is such that

$$\sin \alpha - \alpha^3 + 1 = 0,$$

prove that $0 < \alpha < \dfrac{\pi}{2}$.

**(2)**

(*c*) Use the iteration

$$x_{n+1} = \sqrt[3]{\sin x_n + 1} \quad \text{and} \quad x_0 = 1$$

to obtain the values of $x_1, x_2, x_3, x_4$ and hence find the value of $\alpha$ to 4 decimal places.

**(3)**

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**7.** Given $a > 0$, sketch the graphs of

$$y = ax \quad \text{and} \quad y = |6a^2 - x^2|$$

on the same pair of axes. Indicate clearly the points of intersection of the graphs with the coordinate axes.

**(4)**

(*a*) Show that the point $(2a, 2a^2)$ lies on both the line and the curve.

**(1)**

(*b*) Write down an equation that will give the $x$ coordinate of the second point of intersection of the line and the curve.

**(2)**

(*c*) Solve the equation to find the coordinates of the second point of intersection.

**(4)**

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**8.** (*a*) Express $\mathrm{f}(x) = \sqrt{3}\sin x + \cos x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. The values of $R$ and $\alpha$ are to be given exactly.

**(4)**

(*b*) Hence solve the equation

$$\sqrt{3}\sin x + \cos x = \sqrt{2}, \qquad \text{where} \qquad 0 < x < \pi.$$

**(6)**

(*c*) Sketch the graph of $y = \mathrm{f}(x)$ for $0 \le x \le 2\pi$.

**(4)**

(*d*) You are given that $y = 2\mathrm{f}(x) + 1$. State the maximum and minimum values of $y$.

**(3)**

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**END**          **TOTAL 75 MARKS**